Coherent Curvature Radiation and Proton Counterflow in the Pulsar Magnetosphere

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ABSTRACT

In the proton counterflow model of a pulsar magnetosphere that we have recently proposed, non-relativistic protons are supplied from the magnetosphere to flow toward the pulsar surface and screen an electric field above the polar cap region. In this Letter, we show that the proton counterflow is also suitable for the bunching of pair plasma. The two-stream instability is easily excited and can produce bunches of pairs with a relevant length scale to emit coherent curvature radiation.

Subject headings: instabilities—plasmas—radiation mechanisms: non-thermal — stars: pulsars: general

1. INTRODUCTION

A spinning magnetized neutron star provides huge electric potential differences between different parts of its surface as a result of unipolar induction (Goldreich & Julian 1969). A part of the potential difference may be expended on an electric field along the magnetic field somewhere in the magnetosphere. Although a fully self-consistent model for the pulsar magnetosphere has not yet been constructed, several promising models have been proposed. Among them, the polar cap model (Sturrock 1971; Ruderman & Sutherland 1975) assumes

that an electric field $E_\parallel$ parallel to the magnetic field lines exists just above the magnetic poles. The electric field accelerates charged particles up to TeV energies, and resultant curvature radiation from these particles produces copious electron-positron pairs through magnetic pair production. These pairs are believed to be responsible for radio emission.

The extremely high brightness temperature of pulsar radio emission requires a coherent source for this radiation. In the past decade, various maser processes have been proposed and studied: for example, curvature emission induced by curvature drift or torsion (Luo & Melrose 1992, 1995) and emission due to the cyclotron-Cerenkov and Cerenkov drift instabilities (Lyutikov et al. 1999a,b). In most cases these models are sensitive to the magnetic field strength or other parameters. Although the torsion-driven maser process (Luo & Melrose 1995) is not so sensitive to the physical parameters, we have no definite ideas to achieve the required inverted energy spectrum of particles and twisted field lines. Therefore, there is still no widely accepted model of maser emission. On the other hand, one of the simplest mechanisms of radiation is the coherent curvature radiation from bunches of electron-positron pair plasma, which was discussed mainly in the 1970s.

If N particles are in a bunch of some characteristic size λ, emission with wavelengths longer than λ will be coherent. Then the particles in the bunch radiate like a single particle with charge Ne. The brightness temperature becomes N times that estimated from the intensity when the N single particles radiate incoherently.

Cheng & Ruderman (1977) suggested that the bunching arises from the electrostatic two-stream instability of electron and positron streams. However, Benford & Buschauer (1977) found that the growth time of the instability is far too long. Although some other mechanisms to excite instabilities (e.g. Goldreich & Keeley 1971; Asséo et al. 1980, 1983; Usov 1987) have been proposed, the problem remains unsolved so far.

Another serious problem in the polar cap model is the screening of the electric field. The localized potential drop in the polar cap region is maintained by a pair of anode and cathode regions. For a space charge-limited flow model (Fawley et al. 1977; Scharlemann et al. 1978; Arons & Scharlemann 1979), where electrons can freely escape from the stellar surface, i.e., $E_\parallel = 0$ on the stellar surface, an anode has been considered to be provided by pair polarization. However, as Shibata et al. (1998, 2002) found, the thickness of the screening layer is restricted to be extremely small, in order to screen the electric field consistently. A huge number of pairs must be injected within the small thickness. The required pair multiplication factor per primary particle is enormously large and cannot be realized in the conventional pair creation models.

In addition to the above problems, there are various unsolved problems in pulsar physics;

for example, the current closure problem is a representative one. It may be worthwhile to explore the possibility of a novel model, even if the model has some ambiguities at present. Recently, Asano & Takahara (2004) (hereafter AT) have proposed a new mechanism to screen the electric field. In the AT model, nonrelativistic protons are supplied from the corotating magnetosphere to flow toward the stellar surface. Protons can provide an anode to screen an electric field the standard polar cap model supposes. Injected electron-positron pairs yield an asymmetry of the electrostatic potential around the screening point. The required pair creation rate in this model is consistent with the conventional models.

The existence of the proton counterflows is also favorable for the bunching of pair plasma. The existence of protons apparently makes excitation of two-stream instability easier (Cheng & Ruderman 1980). In this Letter, we show that the pair and proton flows in the AT model excite electrostatic waves, which can explain the pulsar radio emission through coherent curvature radiation. In contrast to most modern theories of pulsar radio emission, the bunch models have been so much discussed from the early days of pulsar research and have received several criticisms, such as insufficient growth rates and rapid bunch dispersion due to random motion of bunching particles (see, e.g., Melrose 1995). However, reconsideration of this old idea without any prejudice may be valuable (Gil et al. 2004) in order to overcome present difficulties in pulsar physics, although the maser models are still an alternative possibility of the radiation mechanism. Another interesting reason to reconsider the bunch model is that space charge density waves appear outside the screening region in the AT model, even though the velocity dispersion of pairs is taken into account (see also Shibata et al. 2002). The two stream instability discussed here may be useful to excite non-linear radiation processes in a relativistic plasma (Lyutikov et al. 1999a,b).

In §2, we show that the two-stream instability is easily excited in the AT model. The excited wavelength is long enough to bunch particles. In §3, we discuss counteraction of the excited waves on the pair flows. §4 is devoted to a summary and discussion.

2. TWO-STREAM INSTABILITY

In this section, we discuss the two-stream instability for the situation the AT model supposes. In the AT model, protons are assumed to flow from the corotating magnetosphere toward the stellar surface. The primary electron beam is accelerated from the stellar surface. Electron-positron pairs start to be injected at a certain height above the polar cap, and the electric field is screened there. Outside this point, two-stream instability may be excited, and the resultant bunching of pair plasma yields coherent curvature radiation.

The AT model requires that the proton current $J_{\rm p}$ is of the order of the Goldreich-Julian (GJ) current $J_{\rm GJ} \equiv -\mathbf{\Omega}_* \cdot \mathbf{B}/2\pi$, where $\mathbf{\Omega}_*$ and $\mathbf{B}$ are the angular velocity of the star and magnetic field, respectively. Hereafter, we assume $\mathbf{\Omega}_* \cdot \mathbf{B} > 0$. In the case of opposite polarity, the electric fields are not screened in this model. There is no observational evidence for two classes of pulsars due to the polarity, as expected. The polarity problem remains unsolved so far for all pulsar models. The proton flow is mildly relativistic (the average velocity in AT $\simeq -0.4c$) in the pulsar frame. The current of the primary electron beam from the stellar surface is also of the order of $J_{\rm GJ}$. The Lorentz factor of pairs at injection is required to be more than ~ 500.

From the above assumptions, we obtain the proton number density $n_{\rm p} \simeq \Omega B/2\pi ce$. Then the proton plasma frequency, $\omega_{\rm pp}^2 \equiv 4\pi e^2 n_{\rm p}/m_{\rm p}$, becomes

$$\omega_{\rm pp}/2\pi \simeq 100 T_{0.3}^{-1/2} B_{12}^{1/2} \quad {\rm MHz}, \tag{1}$$

where $T_{0.3}$ and B_{12} are the rotation period of the pulsar and B in units of 0.3 s and 10^{12} G, respectively.

In order to simplify the situation, we consider one-dimensional homogeneous flows of protons and electron-positron pairs. Since the Lorentz factor of the primary beam of electrons from the stellar surface is too large to contribute to the dispersion relation, we neglect the beam component hereafter. The distributions of protons and pairs are functions of the 4-velocity $u = \beta/(1-\beta^2)^{1/2}$, where $\beta = v/c$. In the linear perturbation theory, the dispersion relation for electrostatic waves (Baldwin et al. 1969) is given by

$$1 + \sum_a \frac{4\pi q_a^2}{\omega m_a} \int du \frac{\beta}{\omega - kv} \frac{\partial f_a}{\partial u} = 0, \tag{2}$$

where f_a, q_a, and m_a denote the distribution function, charge, and mass of the particle species a, respectively. Solutions of the dispersion relation usually yield a complex frequency $\omega = \omega_r + i\omega_i$. The imaginary part ω_i of ω corresponds to the growth rate of waves. A positive growth rate $\omega_i > 0$ implies an exponentially growing wave, while a negative ω_i implies an exponentially damped wave.

In the cold-plasma limit the distribution function of the proton flow may be written as $f_{\rm p} = n_{\rm p}\delta(u)$ in the rest frame of the proton flow. We assume that the number densities and flow velocities of pair electrons and positrons are the same. In this case, the contributions of pair electrons and positrons are degenerate, so that we write the total pair distribution as $f_\pm = n_\pm \delta(u - u_\pm)$. We neglect the injection of pairs, although the pair injection continues outside the polar cap region in general. Then we obtain

$$1 - \frac{\omega_{\rm pp}^2}{\omega^2} - \frac{\omega_{\rm p\pm}^2}{(\omega - kv_\pm)^2 \gamma_\pm^3} = 0, \tag{3}$$

where $\omega_{\rm p\pm}^2 \equiv 4\pi e^2 n_\pm/m_e$, and $\gamma_\pm = 500\gamma_{\pm,5}$ is the Lorentz factor of pairs. The charge density wave due to pairs is constructed by the density waves of electrons and positrons with displaced phases of π.

We normalize k and ω by the proton plasma frequency as $\tilde{k} = ck/\omega_{\rm pp}$ and $\tilde{\omega} = \omega/\omega_{\rm pp}$. We write $\omega_{\rm p\pm} \equiv \xi\omega_{\rm pp}$. The current of the primary electron beam (we have neglected here) is also of the order of $J_{\rm GJ}$. The number of pairs one primary electron creates may be 10^3–10^4. Therefore, $n_\pm/n_{\rm p} \equiv M$ may be in the range of 10^3–10^4. As a result, $\xi = \sqrt{M m_{\rm p}/m_{\rm e}}$ is in the range of 1400–4300.

The condition to yield a pair of complex solutions is given by

$$\tilde{k}\beta_\pm < (1+\zeta)^{3/2}, \tag{4}$$

where $\zeta \equiv \xi^{2/3}/\gamma_\pm$. Since $\gamma_\pm$ is required to be larger than ~ 500 in the AT model, we assume $\gamma_\pm = 500$–1000. In this case, ζ is in the range of 0.1–0.5, and $\beta_\pm \simeq 1$. Thus, the threshold of $\tilde{k}$ is close to unity irrespective of the parameters.

The phase velocity of the excited wave is obtained as

$$\frac{\omega_r}{k} \simeq \frac{v_\pm}{1+\zeta}. \tag{5}$$

The maximum of ω_i is given by $\tilde{k}$ smaller than and close to the threshold $(1+\zeta)^{3/2} \sim 1$. Since the right-hand side of Eq. (5) is of the order of unity, $\tilde{\omega_r}$ for the growing wave is of the order of unity, too. In general, ω_r is much larger than ω_i. Therefore, $\tilde{\omega}_i$ is about ~ 0.1 at most. Numerical solutions (see Fig. 1) confirm this estimate.

The growth time $\sim 1/(0.1\omega_{\rm pp}) \sim 10^{-8}$ s $\ll 1/\Omega$, R/c, where $R = 10^7 R_7$ cm is the curvature radius of magnetic fields, is short enough to bunch particles. Therefore, the bunched particles emit coherent radiation just above the polar cap. The wavelength λ of the excited wave is $2\pi/k \sim 2\pi c/\omega_{\rm pp} \sim 300$ cm. The maximum coherent amplification of curvature radiation by bunches of pairs is obtained at wavelengths long compared to the size of bunches λ. For wavelengths shorter than λ (frequency higher than $\nu_0 \equiv \omega_{\rm pp}/2\pi \sim 100$ MHz), the flux of curvature radiation diminishes with a power law of some index α as $\propto \nu^{-\alpha}$ (see, e.g., Saggion 1975; Benford & Buschauer 1977; Michel 1982) extending to the critical frequency $\nu_{\rm c} = 3\gamma_\pm^3 c/4\pi R \sim 100\gamma_{\pm,5}^3 R_7^{-1}$ GHz. The above frequencies are estimated in the proton rest frame. Since the proton flow is nonrelativistic, the redshift of frequencies by the flow is unimportant. Thus, the coherent radio emission in this model is well consistent with the observed pulsar radio spectra.

The number of pairs N, which can radiate coherently, may be written as $N \simeq \epsilon M n_{\rm GJ}\lambda^3 \simeq 6\times 10^{21}\epsilon(M/10^3)(\lambda_3)^3 B_{12} T_{0.3}^{-1}$, where $\lambda_3 = \lambda/300$ cm, and the factor $\epsilon < 1$ is the fraction

of particles that are in a coherent motion. The cooling time scale for coherent curvature radiation may be written as

$$t_{\rm cool} \simeq \frac{\gamma_\pm m_{\rm e} c^2}{pN} \left(\frac{\nu_{\rm c}}{\nu_0}\right)^{4/3} = \frac{3R^2 m_{\rm e} c}{2e^2 \gamma_\pm^3 N} \left(\frac{\nu_{\rm c}}{\nu_0}\right)^{4/3}, \tag{6}$$

where $p = 2e^2 c\gamma_\pm^4/3R^2$ is the emitting power by a single particle. Here we adopt $\nu_{\rm c}/\nu_0 = 10^3$. If $N > 3\times10^{15}$, $t_{\rm cool}$ becomes shorter than $R/c \sim 3\times10^{-4} R_7$ s. This is realized for $\epsilon > 5\times10^{-7}$ and gives a bright enough radio luminosity. Since high radio luminosity can be realized even for a smaller N if the number of bunches is large enough, this estimate is not unique. On the other hand, the radio brightness temperature $T_{\rm b}$ may be limited by the self-absorption (Cheng & Ruderman 1980); $T_{\rm b} < \gamma_\pm N m_{\rm e} c^2/k_{\rm B}$, where $k_{\rm B}$ is the Boltzmann constant. The value $N = 3 \times 10^{15}$ gives a limit $\sim 10^{28}$ K, which is high enough to be consistent with observations. The above estimate of N and ϵ appear to be appropriate, although it is not unique. Thus, a small value of ϵ suffices for coherent radio emission, although it is hard to estimate ϵ from the first principle because of some nonlinear effects on the bunching process.

3. WAVE COUNTERACTION ON THE FLOWS

The excited waves may produce effective frictional force on the flows. If the frictional force is too strong, the force destroys the structure of the two streams. We have to construct a model that satisfies the two requirements of wave excitation in a short timescale and sustainable structure of the flows, which seems incompatible at first glance. This problem has not been considered seriously so far.

In this section, we obtain a physical requirement to make the frictional force negligible. In the quasi-linear theory, the time-averaged distribution functions evolve according to (see, e.g., Hinata 1978)

$$\frac{\partial f_a}{\partial t} = \frac{\partial}{\partial u}\left[D \frac{\partial f_a}{\partial u}\right], \tag{7}$$

where

$$D = \frac{q_a^2}{m_a^2 c^2} {\rm Re}\left[\int dk \frac{E_k^2}{i(kv - \omega_{r,k}) + \omega_{i,k}}\right], \tag{8}$$

and E_k is amplitude of the electric field with a wavenumber k. Then the change of u due to the friction per unit time is obtained as

$$\dot{u} = \frac{\partial D}{\partial u}. \tag{9}$$

The amplitude E_k is difficult to estimate. The electric field traps pairs, if the amplitude of the excited electric field E becomes larger than a threshold

$$\begin{aligned} E_{\rm max} &\equiv \frac{k\gamma_\pm m_{\rm e}c^2}{2e\gamma_{\rm w}^2} \simeq \frac{\omega_{\rm pp}\gamma_\pm m_{\rm e}c}{2e\gamma_{\rm w}^2} \\ &\simeq 10^3 \left(\frac{\gamma_{\rm w}^2}{10}\right)^{-1} \gamma_{\pm,5} B_{12}^{1/2} T_{0.3}^{-1/2} \quad \text{in esu}, \end{aligned} \tag{10}$$

where $\gamma_{\rm w}$ is the Lorentz factor of the phase velocity of the wave. We have assumed $\gamma_{\rm w}^2 \gg 1$ in the above estimate. However, even if $\gamma_{\rm w}^2 \sim 1$, the correct value differs from the above at most by a factor of 2. In the relativistic limit, $\gamma_{\rm w}^2 \simeq (1+\zeta)/2\zeta$. Thus, $\gamma_{\rm w}^2$ is 10 at most. Assuming the maximum charge density $\rho_{\rm max} \sim E_{\rm max}/\lambda$, we obtain $\rho_{\rm max} \sim \gamma_\pm m_{\rm e}/(\gamma_w^2 m_{\rm p})\rho_{\rm GJ}$, where $\rho_{\rm GJ} \equiv \Omega B/(2\pi c)$ is the GJ charge density. Since $\gamma_\pm \sim 10^3$ and $\gamma_{\rm w}^2 = $ 1-10, the maximum charge density turns out to be of the order of 0.1-1 times the GJ charge density. Although there may be various nonlinear effects to excite waves, the maximum amplitude of the electric field may not be much larger than $E_{\rm max}$.

We approximate the excited electric field by a monochromatic wave of $E_k = E\delta(k - \omega_{\rm pp}/c)$. It is apparent that the time scale for the slowdown due to the friction $t_{\rm fric}$ for the pair flow is longer than that for the proton flow owing to the large value of $\gamma_\pm$. Taking into account $\omega_r \simeq \omega_{\rm pp}$ and $\omega_i \simeq 0.1\omega_{\rm pp}$, $t_{\rm fric}$ for the proton flow is obtained as

$$t_{\rm fric} = \frac{1}{|\dot{u}|} \simeq 5\omega_{\rm pp}\left(\frac{m_{\rm p}c}{eE}\right)^2. \tag{11}$$

We consider that the physical condition remains unchanged within a scale R/c. In order to make $t_{\rm fric}$ be longer than R/c, E should be smaller than $E_{\rm max}$ by a factor of 3. Experience has shown that longitudinal waves are frequently strongly damped by nonlinear processes. So we expect that E may be smaller than $E_{\rm max}$, small enough to keep the fictional force small and strong enough to bunch a small fraction of pair particles.

4. SUMMARY AND DISCUSSION

We examined the two-stream instability and coherent curvature radiation in the proton counterflow model that we have recently proposed. The existence of proton flows is favorable not only for screening of the electric field but also for the bunching of pair plasma by the two-stream instability. This model predicts a high growth rate and wavelength of electrostatic waves appropriate to reproduce observed radio emission by coherent curvature radiation. The growth rate is basically determined by $\omega_{\rm pp} \propto \sqrt{B_{12}/T_{0.3}}$. Since rapidly rotating pulsars tend to have weak magnetic fields, $\omega_{\rm pp}$ is not so sensitive to the model parameters. For

example, $\omega_{\rm pp}/2\pi$ becomes about 50 MHz for $B = 10^9$ G and $T = 1$ ms. On the other hand, for $B = 10^{15}$ G and $T = 10$ s, $\omega_{\rm pp}/2\pi \sim 500$ MHz. The resultant growth rate is also insensitive to the model parameters. It is interesting that the predicted wave length is comparable to the wave length of the space charge density wave, which appears outside the screening region in AT.

We noticed the interesting paper of Lesch et al. (1998) in which they claimed that coherent curvature radiation cannot be the source for the radio emission of pulsars. However, their treatment is based on several simplifying assumptions: a full coherence extending up to $\nu_{\rm c}$, a large coherence volume, and others. Basically, these assumptions lead to a short cooling time and a lower value for the upper limit of luminosity. In our case, the coherence volume is smaller and at $\nu_{\rm c}$ only a partial coherence is supposed. Moreover, only a small fraction of pairs are bunched and the cooling time is much longer than their estimate. Thus, the limit claimed by Lesch et al. (1998) is irrelevant in our case. Examinations of more general constraints are beyond the scope of this Letter.

Although the AT model resolves both the screening and radio emission problems, there remain many ambiguous points: the frictional force discussed in section 3, mechanisms to achieve the proton counterflow, and so on. In the AT model, the proton counterflow comes from the corotating magnetosphere via the anomalous diffusion (Liewer 1985). The currents of the primary beam and protons are most likely to be determined by the global dynamics in the magnetosphere. However, we should not persist in the anomalous diffusion, because there may be other mechanisms to cause the proton counterflow. Considering that we still do not understand well the fundamental issues of pulsar physics, we should be free from any kind of prejudices. We cannot exclude any possibility at the present stage.

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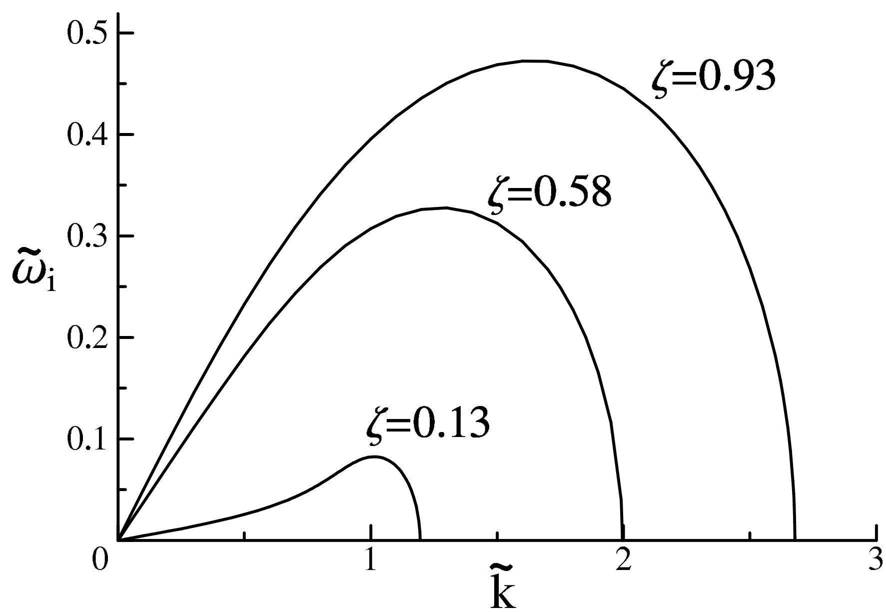

Fig. 1.— Growth rate $\tilde{\omega}_i$ against $\tilde{k}$ for $\zeta = 0.13$ ($\xi = 500$ and $\gamma_{\pm} = 500$), 0.58 ($\xi = 5000$), and 0.93 ($\xi = 10^4$).